# 1.13 Polar and Euler form

**1a.** *[2 marks]*

Let , where .

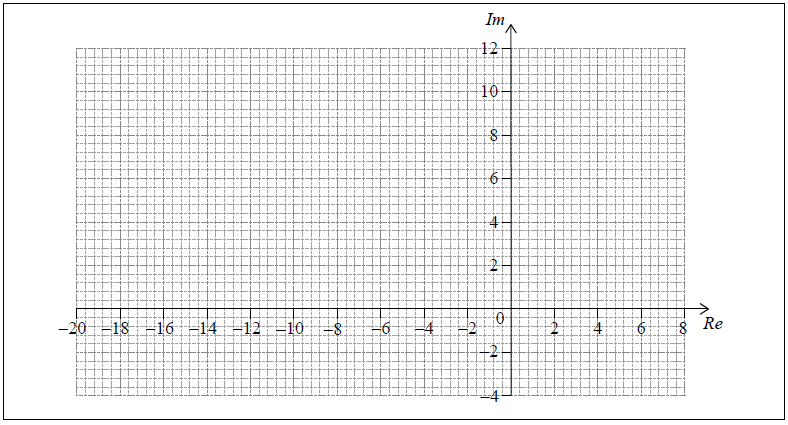
for  = 2,

find the values of , , and .



**1b.** *[3 marks]*

draw , , , and  on the following Argand diagram.



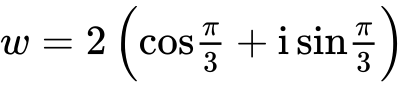
**1c.** *[2 marks]*

Let .

Find the value of  for which successive powers of  lie on a circle.



**2a.** *[3 marks]*

Consider 

Express *w* and *w* in modulus-argument form.



**2b.** *[2 marks]*

Sketch on an Argand diagram the points represented by *w* , *w* , *w* and *w*.



**2c.** *[3 marks]*

These four points form the vertices of a quadrilateral, *Q*.

Show that the area of the quadrilateral *Q* is .



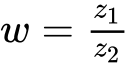
**2d.** *[6 marks]*

Let . The points represented on an Argand diagram by  form the vertices of a polygon .

Show that the area of the polygon  can be expressed in the form , where .



**3a.** *[3 marks]*

Consider the complex numbers  and .

By expressing  and  in modulus-argument form write down the modulus of ;



**3b.** *[1 mark]*

By expressing  and  in modulus-argument form write down the argument of .



**3c.** *[2 marks]*

Find the smallest positive integer value of , such that  is a real number.



**4.** *[5 marks]*

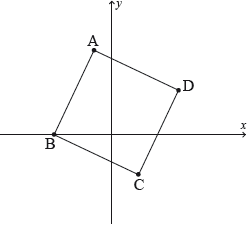
Let .

Hence find the cube roots of  in modulus-argument form.



**5.** *[4 marks]*

In the following Argand diagram the point A represents the complex number  and the point B represents the complex number . The shape of ABCD is a square. Determine the complex numbers represented by the points C and D.



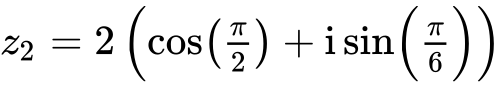


**6a.** *[6 marks]*

Solve the equation  giving your answers in the form  **and** in the form  where .



**6b.** *[11 marks]*

Consider the complex numbers  and .

(i)     Write  in the form .

(ii)     Calculate  and write in the form  where .

(iii)     Hence find the value of  in the form , where .

(iv)     Find the smallest value  such that  is a positive real number.



**7a.** *[6 marks]*

Find three distinct roots of the equation  giving your answers in modulus-argument form.



**7b.** *[3 marks]*

The roots are represented by the vertices of a triangle in an Argand diagram.

Show that the area of the triangle is .



**8.** *[17 marks]*

A geometric sequence , with complex terms, is defined by  and .

(a)     Find the fourth term of the sequence, giving your answer in the form .

(b)     Find the sum of the first 20 terms of , giving your answer in the form  where  and  are to be determined.

A second sequence  is defined by .

(c)     (i)     Show that  is a geometric sequence.

          (ii)     State the first term.

          (iii)     Show that the common ratio is independent of *k*.

A third sequence  is defined by .

(d)     (i)     Show that  is a geometric sequence.

          (ii)     State the geometrical significance of this result with reference to points on the complex plane.

**9.** *[7 marks]*

Consider the complex numbers  and .

(a)     Given that , express *w* in the form .

(b)     Find \* and express it in the form .

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